Cambridge Assessment International Education

Cambridge IGCSE™

ADDITIONAL MATHEMATICS **0606/12**

Paper 1 **May/June 2021**

MARK SCHEME

Maximum Mark: 80

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2021 series for most Cambridge IGCSE™, Cambridge International A and AS Level components and some Cambridge O Level components.

This document consists of **11** printed pages.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1: Marks must be awarded in line with: • the specific content of the mark scheme or the generic level descriptors for the question • the specific skills defined in the mark scheme or in the generic level descriptors for the question • the standard of response required by a candidate as exemplified by the standardisation scripts.
GENERIC MARKING PRINCIPLE 2: Marks awarded are always **whole marks** (not half marks, or other fractions).
GENERIC MARKING PRINCIPLE 3: Marks must be awarded **positively**: • marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate • marks are awarded when candidates clearly demonstrate what they know and can do • marks are not deducted for errors • marks are not deducted for omissions • answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.
GENERIC MARKING PRINCIPLE 4: Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.
GENERIC MARKING PRINCIPLE 5: Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).
GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Maths-Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

MARK SCHEME NOTES

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method marks, awarded for a valid method applied to the problem.

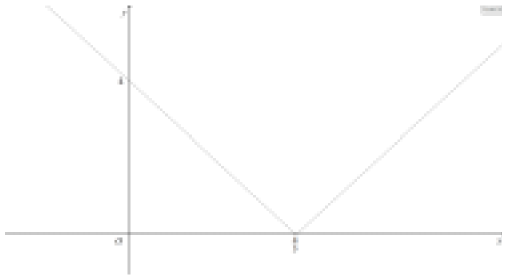
A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. For accuracy marks to be given, the associated Method mark must be earned or implied.

B Mark for a correct result or statement independent of Method marks.

When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. The notation '**dep**' is used to indicate that a particular M or B mark is dependent on an earlier mark in the scheme.

Abbreviations

awrt answers which round to
cao correct answer only
dep dependent
FT follow through after error
isw ignore subsequent working
nfww not from wrong working
oe or equivalent
rot rounded or truncated
SC Special Case
soi seen or implied

Question	Answer	Marks	Guidance
1	$p^0q^{-5}r^{-\frac{2}{3}}$	**3**	**B1** for $a=0$ **B1** for $b=-5$ **B1** for $c=-\frac{2}{3}$
2(a)		**2**	**B1** for symmetrical V shape in the correct quadrant, touching the x-axis. Must have straight lines. **B1** for $x=\frac{4}{3}$ and $y=4$ only, either seen or stated on a modulus graph.
2(b)	$x \leqslant -1$, $x \geqslant \frac{11}{3}$ or 3.67 or better	**3**	**B1** for –1 from a correct method. **B1** for $\frac{11}{3}$ or 3.67 or better, from a correct method.
3(a)	$\overrightarrow{AC} = \mathbf{c} - \mathbf{a}$	**B1**	May be implied
	$\overrightarrow{OP} = \mathbf{a} + \frac{3}{5}\overrightarrow{AC}$ or $\mathbf{c} - \frac{2}{5}\overrightarrow{AC}$	**M1**	Maybe implied, for correct use of ratio $\overrightarrow{OP} = \mathbf{a} + \frac{3}{5}\left(their\,\overrightarrow{AC}\right)$ or $\mathbf{c} - \frac{2}{5}\left(their\,\overrightarrow{AC}\right)$
	$\overrightarrow{OP} = \frac{2}{5}\mathbf{a} + \frac{3}{5}\mathbf{c}$	**A1**	Allow unsimplified
3(b)	$\overrightarrow{OP} = \frac{2}{5}\mathbf{b}$ oe	**B1**	
	$\frac{2}{5}\mathbf{b} = \frac{2}{5}\mathbf{a} + \frac{3}{5}\mathbf{c}$ $2\mathbf{b} = 2\mathbf{a} + 3\mathbf{c}$	**B1**	**Dep** on previous B mark for equating vectors and rearrangement to obtain **AG**
	Alternative $\mathbf{b} = \frac{2}{5}\mathbf{a} + \frac{3}{5}\mathbf{c} + \frac{3}{5}\mathbf{b}$	**(B1)**	Need a clear indication of the method used, in the form of a correct unsimplified statement.
	$2\mathbf{b} = 2\mathbf{a} + 3\mathbf{c}$	**(B1)**	**Dep** for simplification to obtain AG

Question	Answer	Marks	Guidance
4	$\left(\frac{dy}{dx}=\right)\ \frac{1}{2}(3x+2)^{\frac{2}{3}}\quad(+c)$	**M1**	For $k_1(3x+2)^{\frac{2}{3}}$ where k_1 a constant.
	$4=2+c$	**M1**	**Dep** for use of 4 and $x=2$ in *their* $\frac{dy}{dx}$ to obtain c
	$\left(\frac{dy}{dx}=\right)\ \frac{1}{2}(3x+2)^{\frac{2}{3}}+2$	**A1**	May be implied by subsequent integration or by $c=2$
	$y=\frac{1}{10}(3x+2)^{\frac{5}{3}}\quad(+2x+d)$	**M1**	For $k_2(3x+2)^{\frac{5}{3}}$ where k_2 is a constant.
	$6.2=\frac{1}{10}(32)+4+d$	**M1**	**Dep** on previous M1 for use of $x=2$ and $y=6.2$ in *their y*
	$y=\frac{1}{10}(3x+2)^{\frac{5}{3}}+2x-1$	**A1**	Must be an equation
5(a)	$p=16$	**2**	**B1** for $\log_a\frac{5p}{4}=\log_a 20$ oe **B1** for 16, nfww
5(b)	$\left(3\left(3^x\right)-1\right)\left(3^x+3\right)=0$	**M1**	For recognition of a correct quadratic in 3^x and attempt to factorise or use quadratic formula
	$3^x=\frac{1}{3}$ $x=-1$	**2**	**M1 dep** for a correct attempt to solve $3^x=k,\ k>0$ **A1** for one solution only, must be from a correct solution.

Question	Answer	Marks	Guidance
5(c)	$\log_y 2 = \dfrac{1}{\log_2 y}$ or $\log_2 y = \dfrac{1}{\log_y 2}$ or $\log_y 2 = \dfrac{\log_a 2}{\log_a y}$ **and** $\log_2 y = \dfrac{\log_a y}{\log_a 2}$	**B1**	May be implied
	$4(\log_y 2)^2 - 4(\log_y 2) + 1 = 0$ $(2\log_y 2 - 1)^2 = 0$, $\log_y 2 = \dfrac{1}{2}$ or $(\log_2 y)^2 - 4(\log_2 y) + 4 = 0$ $(\log_2 y - 2)^2 = 0$, $\log_2 y = 2$ or $(\log_a y)^2 - 4(\log_a 2)(\log_a 4)\log_a y$ $+4(\log_a 2)^2 = 0$ $(\log_a y - 2\log_a 2)^2 = 0$ $\log_a y = 2\log_a 2$	**M1**	For obtaining a 3 term quadratic equation in either $\log_y 2$ or $\log_2 y$ and solving to obtain $\log_y 2 = k$ or $\log_2 y = k$, may be implied or equivalent using an alternative base.
	$y = 4$	**A1**	nfww
6(a)	$\dfrac{dy}{dx} = 2(3+\sqrt{5})x - 8\sqrt{5}$ or $x = \dfrac{8\sqrt{5}}{2(3+\sqrt{5})}$	**M1**	Either For differentiation must have one correct term. or for use of '$b^2 - 4ac = 0$', so $x = -\dfrac{b}{2a}$ at the stationary point.
	$x = \dfrac{4\sqrt{5}}{3+\sqrt{5}} \times \dfrac{(3-\sqrt{5})}{(3-\sqrt{5})}$ oe leading to $\dfrac{12\sqrt{5}-20}{4}$ oe, this is the minimum acceptable working for this method.	**M1**	**Dep** for equating *their* $\dfrac{dy}{dx}$ to zero with attempt to solve and rationalisation using a two term factor, or rationalisation of $x = -\dfrac{b}{2a}$, using a two term factor with sufficient detail to imply no use of a calculator. Allow multiple equivalents. Allow one numerical slip or sign error.
	$x = -5 + 3\sqrt{5}$	**2**	**A1** for -5 **A1** for $3\sqrt{5}$

Question	Answer	Marks	Guidance
6(b)	$y=\left(3+\sqrt{5}\right)\left(3\sqrt{5}-5\right)^2$ $-8\sqrt{5}\left(3\sqrt{5}-5\right)+60$ $=\left(3+\sqrt{5}\right)\left(45+25-30\sqrt{5}\right)$ $-120+40\sqrt{5}+60$ $=210+70\sqrt{5}-90\sqrt{5}-150$ $-120+40\sqrt{5}+60$	**M1**	For substitution of *their x* and simplification with sufficient detail to imply no use of a calculator. Allow one numerical slip or sign error in the expansion of $\left(3+\sqrt{5}\right)\left(3\sqrt{5}-5\right)^2$ or one sign error in the other terms.
	$=20\sqrt{5}$	**2**	**A1** for all non surd terms = 0 **A1** for $20\sqrt{5}$
7(a)(i)	20160	**B1**	
7(a)(ii)	7200	**2**	**B1** for ${}^6\mathrm{P}_4$ or $6\times5\times4\times3(=360)$ for 'inner' characters or ${}^5\mathrm{P}_2$ or $4\times5(=20)$ for 'outer' characters Must be part of a product
7(a)(iii)	360	**2**	**B1** for ${}^3\mathrm{P}_3$ or 3! or 6 for arrangements of symbols or ${}^5\mathrm{P}_3$ or $5\times4\times3$ (= 60) for the digits Must be part of a product
7(b)	$\dfrac{n!}{(n-5)!5!}=\dfrac{6(n-1)!}{((n-1)-4)!4!}$	**B1**	May be implied by simplification e.g. $\dfrac{n!}{5!}=6\dfrac{(n-1)!}{4!}$ or $\dfrac{n(n-1)(n-2)(n-3)(n-4)}{5!}$ $=\dfrac{6(n-1)(n-2)(n-3)(n-4)}{4!}$
	Simplification of either the numerical factorials or the algebraic factorials	**M1**	
	$n=30$	**A1**	

Question	Answer	Marks	Guidance
8(a)	$\lg y = b\lg x + \lg A$	B1	May be implied by subsequent work
	$4.37 = 5.36b + \lg A$ $0.57 = 0.61b + \lg A$	M1	For at least one correct equation
	$b = 0.8$	A1	
	$\lg A = k \quad (0.082)$ $A = 10^k$	M1	**Dep** for substitution to obtain $\lg A = k$ and hence A
	$A = 1.21$	A1	
	Alternative 1 $\lg y = b\lg x + \lg A$	(B1)	May be implied by subsequent work
	Gradient = $\frac{4.37-0.57}{5.36-0.61}$	(M1)	
	$b = 0.8$	(A1)	
	$\lg A = k \quad (0.082)$ $A = 10^k$	(M1)	**Dep** for substitution into a correct equation to obtain $\lg A = k$ and hence A
	$A = 1.21$	(A1)	
	Alternative 2 $10^{4.37} = A \times 10^{5.36b}$ or $10^{0.57} = A \times 10^{0.61b}$	(B1)	
	$3.8 = 4.75b$	(M1)	For eliminating A correctly Must have B1.
	$b = 0.8$	(A1)	
	$A = 10^{4.37-(5.36\times(their\, b))}$ oe	(M1)	For a correct attempt to find A. Must have B1
	$A = 1.21$	(A1)	
8(b)	$y = 1.21(3)^{0.8}$ or $\lg y = 0.8\lg 3 + 0.082$	B1	**FT** for substitution into *their* equation
	y = awrt 2.9	B1	
8(c)	$3 = 1.21x^{0.8}$ or $\lg 3 = 0.8\lg x + 0.082$	B1	**FT** for substitution into *their* equation
	x = awrt 3.1	B1	

Question	Answer	Marks	Guidance
9(a)	$d = 12$	**B1**	
	$\frac{n}{2}\left(-8+(n-1)12\right) > 2000$ $3n^2 - 5n - 1000 > 0$	**M1**	For use of sum formula to obtain a three term quadratic inequality or equation
	$n = \frac{5 \pm \sqrt{25+12000}}{6}$ $n = 19.1$	**M1**	**Dep** for attempt at critical value(s) using *their* quadratic, may be using a calculator, so may be implied by a correct answer of 20.
	$n = 20$	**A1**	
9(b)(i)	$r = 3$	**2**	**M1** For $ar^6 = 27$ and $ar^8 = 243$ with an attempt to eliminate a to obtain r^2. Allow other valid methods.
9(b)(ii)	3^{26}	**2**	**B1** for $a = \frac{1}{27}$ or 3^{-3} nfww
9(c)	Common ratio or $r = \sin\theta$	**B1**	May be implied by e.g. $\frac{1}{1-\sin\theta}$ or $\frac{1-\sin^n\theta}{1-\sin\theta}$
	$-1 < \sin\theta < 1$ or $\lvert\sin\theta\rvert < 1$ or $-1 < r < 1$ or $\lvert r\rvert < 1$ with no incorrect statements seen.	**B1**	**Dep** on previous B1
10(a)	$\frac{1}{\sin\alpha} + \frac{1}{\cos\alpha} \quad (=0)$	**B1**	For dealing correctly with $\operatorname{cosec}^2\alpha$ and $\sec^2\alpha$ to obtain an expression in $\sin\alpha$ and $\cos\alpha$ only
	$\tan\alpha = -1$ or $\sin\alpha = -\cos\alpha$	**B1**	For an equation in $\tan\alpha$, may be implied by a correct solution.
	$\alpha = -\frac{\pi}{4}$ or -0.785 $\alpha = \frac{3\pi}{4}$ or 2.36	**2**	**B1** for one correct solution **B1** for a second correct solution and no extra solutions in the range.

Question	Answer	Marks	Guidance
10(b)(i)	$\dfrac{\cos^2\theta+1-2\sin\theta+\sin^2\theta}{\cos\theta(1-\sin\theta)}$	**M1**	For dealing with the fractions correctly and expansion of $(1-\sin\theta)^2$
	$\dfrac{1+1-2\sin\theta}{\cos\theta(1-\sin\theta)}$ or better	**M1**	**Dep** for use of identity, may be implied by $\dfrac{2(1-\sin\theta)}{\cos\theta(1-\sin\theta)}$
	$\dfrac{2(1-\sin\theta)}{\cos\theta(1-\sin\theta)}$	**M1**	**Dep** on previous M mark for simplification
	$\dfrac{2}{\cos\theta}=2\sec\theta$	**A1**	Need to see this detail for A1 Need to have had θ in every trigonometric ratio.
	Alternative 1 $\left(\dfrac{\cos\theta}{1-\sin\theta}\times\dfrac{1+\sin\theta}{1+\sin\theta}\right)+\dfrac{1-\sin\theta}{\cos\theta}$	**(M1)**	
	$\dfrac{\cos\theta(1+\sin\theta)}{\cos^2\theta}+\dfrac{1-\sin\theta}{\cos\theta}$	**(M1)**	**Dep** for use of identity
	$\dfrac{1+\sin\theta}{\cos\theta}+\dfrac{1-\sin\theta}{\cos\theta}$	**(M1)**	**Dep** on previous M mark for simplification
	$\dfrac{2}{\cos\theta}=2\sec\theta$	**(A1)**	Need to see this detail for A1 Need to have had θ in every trigonometric ratio.
	Alternative 2 $\dfrac{(1-\sin^2\theta)+(1-\sin\theta)^2}{\cos\theta(1-\sin\theta)}$	**(M1)**	For dealing with the fractions and using $\cos^2\theta=1-\sin^2\theta$.
	$\dfrac{(1-\sin\theta)(1+\sin\theta)+(1-\sin\theta)^2}{\cos\theta(1-\sin\theta)}$	**(M1)**	**Dep** for factorising $1-\sin^2\theta$
	$\dfrac{1+\sin\theta+1-\sin\theta}{\cos\theta}$	**(M1)**	**Dep** for simplification
	$\dfrac{2}{\cos\theta}=2\sec\theta$	**(A1)**	Need to see this detail for A1 Need to have had θ in every trigonometric ratio.

Question	Answer	Marks	Guidance
10(b)(ii)	$\cos 3\phi = \frac{1}{2}$	**B1**	
	$\phi = 20^\circ, 100^\circ, 140^\circ$	3	**M1** for one correct solution of *their* $\cos 3\phi = k$ using a correct order of operations **A1** for 2 correct solutions **A1** for a third correct solution with no extra solutions in the range
11	$\frac{dy}{dx} = \frac{(2x-3)\frac{2x}{x^2+2} - 2\ln(x^2+2)}{(2x-3)^2}$	3	**B1** for $\frac{2x}{x^2+2}$ **M1** for differentiation of a quotient
	When $x = 2$, $\frac{dy}{dx} = \frac{4}{6} - 2\ln 6$, –2.92 Gradient of normal = 0.3428	**M1**	For $-\frac{1}{their\ \frac{dy}{dx}}$
	When $x = 2$, $y = \ln 6$ or $1.79(176)$	**B1**	
	Equation of normal: $y - \ln 6 = -\frac{1}{their\ \frac{dy}{dx}}(x-2)$ or $\ln 6 = -\frac{1}{their\ \frac{dy}{dx}} \times (2) + c$	**M1**	**Dep** for equation of normal using $-\frac{1}{their\ \frac{dy}{dx}}$ and *their* y with $x = 2$.
	When $x = 0$, $y =$ awrt 1.11	**A1**	Must be evaluated.